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enter into any formula whatever. That which a formula calculates is a certainty; and with the act of the mind by which the certainty is accepted in lieu of a contingency, and the nature of things by which events come to justify such acceptance, algebra and its processes have nothing to do. Because the probability of an event is *three tenths*, we hold *three tenths* of the benefit which such event will bring with it if it should happen, to be the equivalent certainty. We may be right or wrong, and we know that we are right; but, right or wrong, the assignment of  $av + bv^2 + cv^3 + \&c.$  as the value of a life annuity is the assignment of the annuity certain  $a, b, c, \&c.$  as a composition for the contingency.

We are in the habit of saying that the value of an event multiplied by the chance of its happening is the *value of the chance*: it would be better, in some respects, to call this product the *equivalent certainty*. Such a phrase, had it been adopted from the beginning, would perhaps have prevented that utter separation which has taken place between the cases in which  $av + bv^2 + \&c.$  represents a life annuity, and that in which it represents an annuity certain.

Among the preceding solutions is that of a problem proposed in the last Number (p. 243). One solution was forwarded to the Editor; but it did not fulfil the required conditions. For instance, the summation of the following "series involving powers,"  $1 + r + \dots + r^{n-1}$ , was effected by multiplying it by  $1-r$ , and thus reducing it to  $1-r^n$ . Between this and the theorem  $(1-r^n) : (1-r) = 1 + r + \dots + r^{n-1}$  there is but difference of form.

*On a simple plan of Classifying the Policies of a Life Assurance Company, so as to possess, at any time, the means of forming a Table of the Mortality experienced in the Office.* By SAMUEL BROWN, F.S.S., One of the Honorary Secretaries of the Institute of Actuaries, and Actuary of The Mutual Life Assurance Society.

[Read before the Institute 27 March, 1854, and ordered by the Council to be printed.]

AFTER the valuable essays with which the members of this Institute have been favoured by our learned Vice-Presidents and other gentlemen on the subjects of our professional studies, I feel that an apology is due to you for introducing a topic of apparently so humble a character; but I have been gratified to perceive, that many of the papers read before you, and especially those above referred to, have been of such a practical character as to prove

incontestibly the utility of this Institute as the means of adding to the experience of every member, and simplifying the business details as well as the theory of life assurance.

Mr. Jellicoe's paper on the experience of the Eagle Insurance Office, which led to such an interesting discussion at a previous Sessional Meeting, recalled to my mind the toilsome labours in which for more than a twelvemonth, when in the Equitable, I was engaged, under Mr. Morgan, in bringing together the materials for the Equitable experience, printed in 1834, and in forming tables deduced therefrom. The number of books, which had to be consulted for a period of seventy years, to ascertain the exact date and mode in which the members (many having several policies effected on their lives) ceased to be connected with the Society, can only be conceived by those who have had to gather the same materials from books not arranged to convey the information sought for. The dust of oblivion had to be brushed away from the name of many a totally forgotten member, that he might form a useful particle in a new collection of facts, intended to instruct and benefit a new generation.

Mr. Arthur Morgan, in the introduction to his *Tables*, remarks—"From the time when the Equitable Society was first established until within the last few years, it certainly was never contemplated by those who were engaged or consulted in conducting its affairs, that the probabilities of life amongst its members would afford sufficient data for constructing tables of the expectations of the duration of human life, or of the values of life annuities, which might be safely available for any practical purposes; nor were the accounts arranged with a view to the exhibition of a distinct statement of the duration of the lives assured at each particular age, as displayed in the following tables, although an annual comparison of the probabilities of life in the Society with those in the Northampton Tables has been regularly made, in conformity with a plan suggested by Dr. Price to the court of directors in the year 1776." No doubt the same trouble and difficulty have been experienced by the managers of the other Assurance Companies, who have with so laudable a liberality contributed to our knowledge on the same important subject.

Mr. Galloway, also, in his introduction to the *Tables of Mortality, deduced from the Experience of the Amicable Society*, observes—"In collecting the materials for these tables, great pains were taken to insure the most scrupulous accuracy. In the early part of the period over which the observation extends, the Office

registers were not arranged with a view to the present investigation; and as a large proportion of members were assured in more than one policy, while the record of the second assurance did not always contain a reference to the first, much difficulty was experienced in ascertaining the exact number of persons who had stood the chances of life in the Society. After repeated comparisons of different sets of books, and reference to the money accounts in all cases of discrepancy, a list, correct (I believe) in every respect, was at length prepared, containing the following particulars:—1, The name of each individual on whose life an assurance was effected for the first time; 2, his office age at admission; 3, the date of his admission; 4, the date of his death or discontinuance, if not living in the Society on the 5th day of April, 1841: which are all the data required for the construction of the tables. The ages at admission were certified by abstracts from baptismal registers, or by affidavits, and the dates were carefully checked.”

I happen to be in possession of an older and very elaborate table, prepared by Mr. Pensam, the former registrar, showing the mortality of the Amicable Society from 1731 to 1831, with the number of assurances, and the deaths, and a comparison with the Northampton Table at every age of entry and for every year after admission; together with the separate experience from 1807 to 1831, prepared in the same manner. I possess also the Total Experience Tables, showing the law of mortality in the same Society during the century preceding 5th April, 1831, with various arithmetical tables deduced therefrom, prepared by Mr. Finlaison. But both these tables appear to have been superseded, and the trouble taken in forming them rendered useless, by the more accurate and laborious inquiries of Mr. Galloway. I merely mention these facts to show the great saving of time, labour, and expense, which will result by the simple arrangement for collecting these facts from the beginning of a Society, which I shall have to propose.

Further: it may be gathered, from the introduction to the *Tables exhibiting the Law of Mortality deduced from the combined Experience of Seventeen Life Assurance Offices*, printed in 1843, that not only was the task to each contributing Company one of great labour and difficulty, but that the method of registering entries and ages was so different, that some of the returns, after the time and toil expended, were quite useless for the purpose required, notwithstanding the willingness of the managers to communicate their experience and the value of the information asked for. The former we may gather from the length of time the investigation

occupied; for the circular with the forms was dated as sent out on the 25th Sept., 1838, and the Committee remark, "that the whole of the returns were not received from the contributing Offices until after the middle of the year 1841, and that the diversified classification of the assurances has added much to the complexity of the operations." In another part they state that "it was found impracticable to combine the facts contained in the whole of the returns, so as to show the exact number that died, and were discontinued, and existing at each age, as some of the returns were made up giving the current age, and others the completed age, at entering; and the number existing in some returns had been exposed to the risk of mortality for a whole year, and in others to no part of it."

I need not spend much time in impressing upon you, gentlemen, the value of the information thus sought to be obtained; it will be evident to every member of this Institute. We must all feel, too, the credit which is due to the compilers of each of the elaborate collection of tables I have enumerated, for the zeal and liberality of mind which have led them to suggest and prosecute such toilsome investigations, and to procure their publication for the benefit of all our profession. As one who has profited by their scientific labours, I gratefully record my own sense of the honour which is due to them; and I would venture respectfully to urge upon every member of the Institute, that an example so honourable ought to be followed by everyone, who, in his sphere, can add something to our own experience—something which may tend to the eventual benefit of his country. It is by this public spirit that we shall not only give to our profession that position, both before the Government and the public, which everyone who has chosen it ought to seek to achieve for it, but we shall prove also, that those who love science for its own sake cease to be actuated by merely selfish motives, and set before themselves a more noble and intellectual distinction as the aim and object for which they toil.

The inquiries here suggested are peculiarly suited for the members of this Institute. There must always be a difficulty in bringing together for a special occasion a number of individuals to work harmoniously for a common object. This spirit, at least, we profess to have already acquired; and the frequent and interesting discussions which we have together will teach us how best to employ it. Since the publication of the tables described, an immense and valuable collection of facts has been accumulating in the records of the different Offices, from which many important

results might be deduced, and the law of mortality in various classes of the community probably obtained. I have drawn attention to the difficulty of collecting the facts already published, because it is easy to foresee that the same excuses which were made before will probably be made against furnishing similar information hereafter; yet I venture to affirm that its publication will be much more needed, from the changes in progress in the working of our business, the new combinations to which life assurance is applied, and the entirely new classes of society who come within the sphere of its operations. The immense advantages which would be derived from a combination of the materials accumulating, have induced me to point out what appears to me a very simple plan of classifying the policies of a Life Assurance Company from its commencement, so that all the important facts, by one entry, will be preserved in a form for future reference, and even for being united with those of any other Company, with nothing more than a mere summary of results, which can be made at any time when it is required. It is very important to notice, that some of the columns are almost absolutely necessary if a valuation of the liabilities is to be made frequently, and that the plan proposed will enable it to be performed from year to year with the mere labour of calculating the values of the total sum assured and premiums in force at each age.

I here present the form of the book; and after describing the use of the columns, I will conclude by pointing out the facilities and advantages which it offers.

#### FORM OF CLASSIFICATION BOOK.

##### *Left-hand Page.*

| Age.* | (1)<br>Date of<br>Admission. | (2)<br>How<br>cancelled. | (3)<br>Duration<br>of Policy. | (4)<br>Policy<br>No. | (5)<br>Sum. | (6)<br>Premium. | (7)<br>Sex.<br>(F.) | (8)<br>Occupation. | (9)<br>Remarks. |
|-------|------------------------------|--------------------------|-------------------------------|----------------------|-------------|-----------------|---------------------|--------------------|-----------------|
|       |                              |                          |                               |                      |             |                 |                     |                    |                 |

##### *Right-hand Page.*

| (1)<br>Date of<br>cancellation. | (2)<br>How<br>cancelled. | (3)<br>Policy No. | (4)<br>Sum<br>assured. | (5)<br>Premium. | (6)<br>Cause of Death. | (7)<br>Remarks. |
|---------------------------------|--------------------------|-------------------|------------------------|-----------------|------------------------|-----------------|
|                                 |                          |                   |                        |                 |                        |                 |

\* Date of birth may be here inserted in another column, if necessary.

Col. 1, in the left-hand page, is the date of entry into the Society, which should comprise both the day and month. Col. 2 is the initial of the letter how cancelled—(C), claim; (S), surrendered; (F), forfeited; (E), expired. The last three initials are comprised under the head of “discontinued” in the tables to which we have before referred, but they should be distinguished against each policy. Col. 3, the important entry of the actual duration of the risk, which should be in years and decimals of a year from the actual day of admission. Col. 4 contains the policy number; 5, the sum assured; 6, the premium; 7, the sex, marked by a single letter, only when the assurance is on a female life; 8, the occupation; and a column (9) is left blank for remarks. In this may be noted, if required, the period during the existence of the policy for which the life assured has been exposed to the risk of foreign residence, or any other unusual risk, and when it commenced and terminated.

On the opposite page we have Col. 1, the date when the policy was cancelled. Both the day and the month should be given. In case of death, the entry in this column should be the day of death as proved by the certificate; in case of surrender, the day on which the value was paid; in all cases, indeed, the day on which the risk to the Society actually terminated. Col. 2 contains the mode in which it was cancelled, marked by an initial as before; Col. 3, the policy number; Col. 4, the sum assured; Col. 5, the premium paid; Col. 6, the disease of which the party died, in case the policy became a claim; and a column (7) is left for remarks.

For the sake of simplicity, let us assume that the book is about to be commenced at the end of the first year of the Society’s operations. The different pages are headed with the different ages of life, allowing in consecutive pages for as many entries as are likely to be made at each age of admission, increasing by one year for as many years as the book is intended to last. Each policy is to be entered in the order of its number on a separate line, with the details required under the different columns. The age to be taken is the Office age, and which, as the parties may in most Companies be supposed to enter one with another at half a year short of this age, and quit the Society one with another half a year short of their next Office age, when they leave, will allow the comparisons of living and dying to be made at very nearly the true numeral ages of life. On the other side of the account, we write off under the proper heading the items of policies cancelled before the end of the first year. Deducting this side from the other, we leave what may

be called the balance of the numbers, sums, and premiums on the total policies in force at each age: in other words, we have all the materials ready for the valuation of the Society's liabilities at the end of the first year. The sum of these should be carefully checked by the sum of the entries into the policy ledger, with the cancelled policies first deducted, so that no error will escape detection, except the chance of the entries being made under wrong ages. Proper care should be taken to ensure accuracy in this respect, but the errors in the valuation (if any) arising from this source will be very trifling, as they would probably only consist of the difference between the value of a policy at one age and the next. I look upon this practical use of the book, in the form I suggest, as very important, since every actuary will feel the satisfaction of knowing approximately from year to year the actual working of the Society and the growth of its profits, even though no use should be made of the information till the period of division. If it should be necessary even to value every policy separately, as in some of the Offices is still the practice, it is nevertheless very convenient to be able to check the results by a method which, in a valuation of several millions, would not give a difference of more than a few hundreds of pounds.

At the end of the next year we begin by entering on the right-hand side the cancelled policies in the same form as in the preceding year, at an Office age one year older than before, as on the other side all which stand as existing policies before they are dealt with in this way are one year older than they stood at the preceding date. Part of policies surrendered, and any cancelment of premiums only, by commutation or otherwise, must be brought into the proper column without touching the numbers of the policies. Having deducted from the left-hand side the total of the numbers, sums, and premiums written off, it only remains for us to enter beneath each policy which has been effected at the Office age at which the remainder now stand, with any increase in the sums or premiums of the old policies, and we have again ready for valuation at the end of the second year all the policies in the Society existing at an age one year older than in the entries of the preceding year. I need not pursue this process further. It is evident that, however long the Society may have been in existence, the last result of the continuation of this page, so obtained, will be the amount for valuation under the age last arrived at. Thus, to assume the existing age of 50, in 20 years from the commencement of the Office, it will be the remainder of the policies which have been admitted respectively in each Office year at 30, 31,

32, &c., down to 50 years of age, and which are then existing at the last-mentioned age. No mistake can occur in cancelling under the wrong age, because the very policy which has been entered on the left hand will, with the identical sum and premium, and any changes effected in them, have to come off on the right-hand page. A moment's inspection will show where the difference, if any, arises. If any excess of premium is taken for any unusual risk, the ordinary premium should be charged in the premium column, and the excess in another column by the side; as this will afford the means, when comparing the mortality, of seeing whether the increase of premium is of an equal per centage with the increase of mortality in these classes. It may be proper to mention, that a column may be provided for the reversionary bonus, where this is ascertained at long intervals; and another for the reduction of premium, if the bonus be taken in this way. But I prefer keeping these particulars in a book, of a similar form as to the ages, but which, being without any other column than the number of the policy, will allow both these particulars to be entered, compared, and valued for several years successively.

I will now notice in a few words the facility that a book in the form suggested affords for composing a table of mortality at any given time. For this purpose, the columns actually required are the date of admission, the mode of cancelment, and the duration of the policy. Assuming, as in the previous example, that if all the policies in that page or continuous pages of entry were in force, they would represent members of 50 years of age, we have only to enter, on a sheet of paper properly ruled, the number existing, discontinued, and died at each successive age, by counting the number existing at the date of observation, the number which were written off under either head having more than 19 years' and less than 20 years' duration, the number having more than 18 and less than 19 years' duration, and so on to less than one year duration, and arrange under each other such as entered at the same ages in the different years since the establishment of the Office. In order to distinguish policies from lives, we may enter the name or occupation of the party under the first policy he effects; and on the lines headed "occupation" in which his subsequent policies appear, refer by number to each of the policies existing on his life. In all calculations relating only to lives, only those policies in which the name or occupation is entered need count, taking care that the duration of membership is marked in lieu of the duration of the first policy only.

If it be desired to separate the investigation into the mortality amongst "male or female lives," or according to "occupation," or the actual experience of an Office as to its "policies," as distinguished from "lives assured," *age* will still be the element of all the inquiries; and the only deviation necessary from the former process will be to carry out the facts, which we have considered as general, into separate tables. The sum of their results should be identical in number of facts with those which would be obtained in the manner I propose under the head of "lives" or "policies."

I need scarcely allude to the many advantages which would result from the general adoption by every Office of a book in this form. ' The labour of one clerk for a few days would enable the entries to be kept up from year to year with perfect correctness; and should the Offices agree at any time to unite their experience, and ascertain in what respect different classes of assured lives vary in mortality from the average of the country at large, there would be no need of an elaborate research into old books framed for a totally different purpose. A simple summation of results and transmission of them to this Institute, or a Central Committee of Actuaries, would, without communicating to strangers the internal condition or management of a Company, allow the reasonings of mathematical men to be brought to bear on results, without the labour and waste of time in collecting them, and science and business might unite in a common purpose for the public good.

Considering that the engagements of Life Assurance Companies are variously computed at from £150 to £200,000,000, we can hardly overrate the importance of knowing the laws of mortality and disease on which our calculations depend. The valuable and elaborate essays of Dr. Farr, in the Registrar General's Reports, should act as a stimulus to every member of our profession to throw such light as his experience will enable him on the great questions which Dr. Farr has so ably discussed. We not only are deeply interested in knowing in what respects the mortality amongst assured lives corresponds with or differs from that of the public at large; but also, whether it may not from time to time undergo changes from new methods of carrying on business, or new classes who may be brought in to partake of the benefits of life assurance.

I will now conclude with suggesting that every Company should at once commence from this time a record somewhat in the form proposed. The past may be made up, as opportunities

of leisure occur. I have acted under the impression of the future value of such a work in my own Office, and have just completed the classification of the policies from the beginning; and it is satisfactory to state, that the results, after additions and deductions for 20 years, exactly agree with the particulars of the policies in force, though in the course of that time many minute alterations of premiums, ages, &c., must have occurred. Those who feel the value of time will not despise even slight suggestions, such as I have ventured to lay before you. If they help to remove the serious difficulty to combining the experience of several Assurance Companies—viz., the labour and time which cannot be spared for bringing the first materials together—they will, I am convinced, lead to results hereafter which not only the actuary but the public will benefit by. Simplicity is often the effect of experience; and whilst other animals may by instinct work to perfection, man must improve by the communicated knowledge of another, and profit even by his mistakes and failures. It is this feeling which has induced me to bring before you a plan which I have found by experience to be useful, which is simple in arrangement, capable of being used for other official purposes, and which, with very little addition to present labour, may afford the means hereafter of making new discoveries in the science of probability and the statistics of life.

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*On the Method of Approximating to the Values of Deferred and other Life Annuities, when payable Half Yearly and Quarterly.*  
By HOLMES IVORY, one of the Vice-Presidents of the Institute of Actuaries.

[Read before the Institute 24 April, 1854, and ordered by the Council to be printed.]

THE difference between the value of an immediate life annuity when payable yearly, and that of the same annuity when payable half yearly or quarterly, has, as is well known, been variously estimated by our leading authorities.

According to Dr. Price and Mr. Morgan, “a fifth of a year’s purchase will be generally more than a sufficient addition if the value of the annuity is desired payable half yearly, and three tenths of a year’s purchase if the value of the annuity is desired